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# CRI TECHNOLOGY DIGEST

**CEMENT RESEARCH INSTITUTE OF INDIA**

## RATIONALIZATION OF DESIGN OF MACHINE FOUNDATIONS IN CEMENT PLANTS

### Part II Approximate Analysis

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### INTRODUCTION

Foundations provided for different machinery and equipment in a cement plant are, in general, subjected to dynamic loads. It is, therefore, essential to investigate their dynamic behaviour for arriving at an economical, safe and rational design. Various types of machinery and foundations in a cement plant were briefly discussed and a computer based method was presented in Part I*, for the dynamic analysis of block foundations taking into account the effects of damping, model coupling, embedment depth and in-phase soil mass. Part II presents simplified design procedures for the determination of degree of coupling and equivalent viscous damping to be used in the dynamic analysis of machine foundations. The suggested procedures, comprising simple formulae and charts, yield quick and accurate results. Dynamic analysis of a typical crusher foundation is also presented to illustrate the use of the suggested procedures.

### DETERMINATION OF THE DEGREE OF COUPLING

A block foundation subjected to a two-dimensional dynamic loading will respond in three modes of vibration, namely, horizontal (sliding), vertical and rotational (rocking). While it is easy to decouple the vertical mode of vibration by suitably adjusting the layout dimensions of a block foundation, significant coupling may exist between horizontal and rotational modes, particularly when the machine is located high above the base.

Analysis of block foundations with coupled modes of vibration is quite tedious especially when damping is included. It is, therefore, imperative for a designer to know in the initial design phase itself that whether a coupled mode analysis is necessary in a given situation or not.

The extent of coupling depends on several parameters, such as geometrical dimensions of the foundation, eccentricity between point of

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application of load and the centre of gravity of the system; and soil characteristics underlying the foundation block. A study of the equations of motion of a block foundation shows that the extent of coupling between rocking and sliding modes is mathematically related to the height (S) of the centre of gravity above the base and uncoupled frequencies $\omega_\theta$ and $\omega_x$ in rotational and sliding modes respectively. A parameter $\Psi$ referred to as the degree of coupling is expressed in eq (1) as a function of (i) dimensionless variable $\delta$ (the ratio S/r, where r is the radius of gyration about the axis passing through the centre of gravity and perpendicular to the plane of the paper), and (ii) frequency ratio $\omega_\theta/\omega_x$.

$$\psi = \frac{2\delta}{(\delta^2-1) + \left\{\frac{\omega_\theta}{\omega_x}\right\}^2 \pm \sqrt{\left\{\delta^2 + 1 + \frac{\omega_\theta^2}{\omega_x}\right\}^2 - 4\left\{\frac{\omega_\theta}{\omega_x}\right\}^2}} \qquad \text{......... (1)}$$

Fig 1 shows graphically the relationship between $\psi$, $\delta$ and $\omega_\theta/\omega_x$. This will enable a designer to estimate the degree of coupling quantitatively for a given value of $\delta$ and frequency ratio $\omega_\theta/\omega_x$.

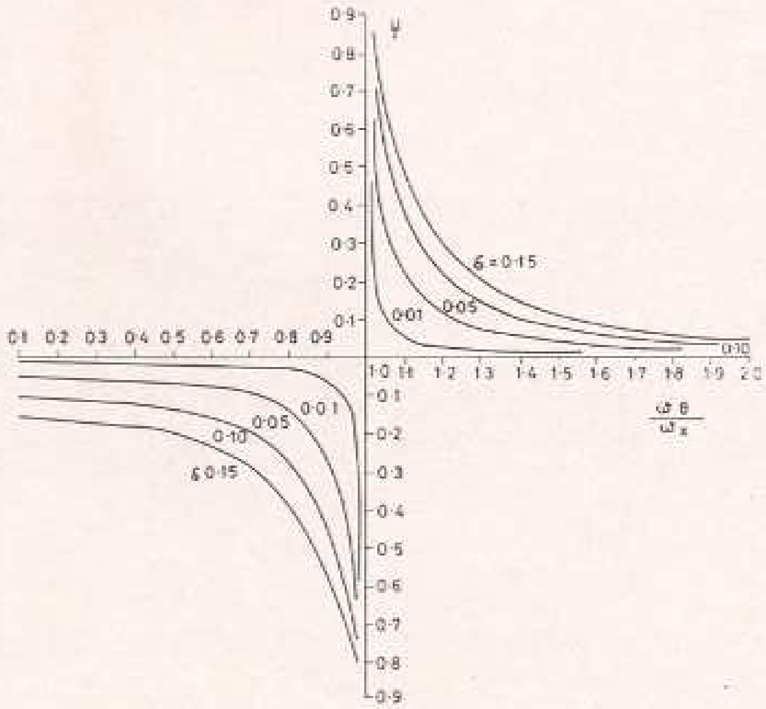


*Fig 1 Frequency ratio vs degree of coupling*

## DETERMINATION OF EQUIVALENT VISCOUS DAMPING

Damping in soil foundation system can be represented by two components: (i) viscous damping representing the energy lost in radiation, and (ii) hysteretic damping representing the energy lost in loading cycles on the material. It can be seen from Fig 2 that the responses of a single degree of freedom system for viscous damping and hysteretic damping are reasonably close for small values of damping coefficients. Therefore, for design purposes, hysteretic damping coefficient can be treated as equivalent viscous damping coefficient and can be directly added to the viscous damping component to give the total damping of the system. For example if the hysteretic damping coefficient D is 0·06 and the viscous damping coefficient of soil $\beta$ is 0·30, the total damping of the soil can be taken as viscous with coefficient equal to 0·36. The same concept can be extended to find modal damping required when the modes are coupled. This has been demonstrated in the analysis of a crusher foundation presented in the following.

**TABLE 1 DESIGN DATA**

| | | | |
|---|---|---|---|
| A | **Machinery Details** | | |
| | Total weight of the machine with motor ($W_m$) | = | 33·3 t |
| | Weight of the moving crushing plate ($W_b$) | = | 4·36 t |
| | Weight of pendulum ($W_c$) | = | 1·7 t |
| | Weight of eccentric ($W_o$) | = | 1·3 t |
| | Eccentricity (r) | = | 0·03 m |
| | Counter weight ($W_d$) | = | 0·05 t |
| | Eccentricity of counter weight = ($r_1$) | = | 0·725 m |
| | Operating speed N | = | 260 rpm |
| B | **Soil Parameters** | | |
| | Soil density (v) | = | 1·8 t/m$^3$ |
| | Shear modulus (G) | = | 10·0 × 10$^3$ t/m$^2$ |
| | Poisson ratio ($\upsilon$) | = | 0·35 |
| | Viscous damping ratio ($d_{\theta i}$) | = | 0·06 |
| C | **Size of the Block Foundation** | | |
| | Length | = | 6·30 m |
| | Breadth | = | 3·50 m |
| | Height | = | 2·74 m |
| | Height of the CG of the system above the base | = | 2·0 m |

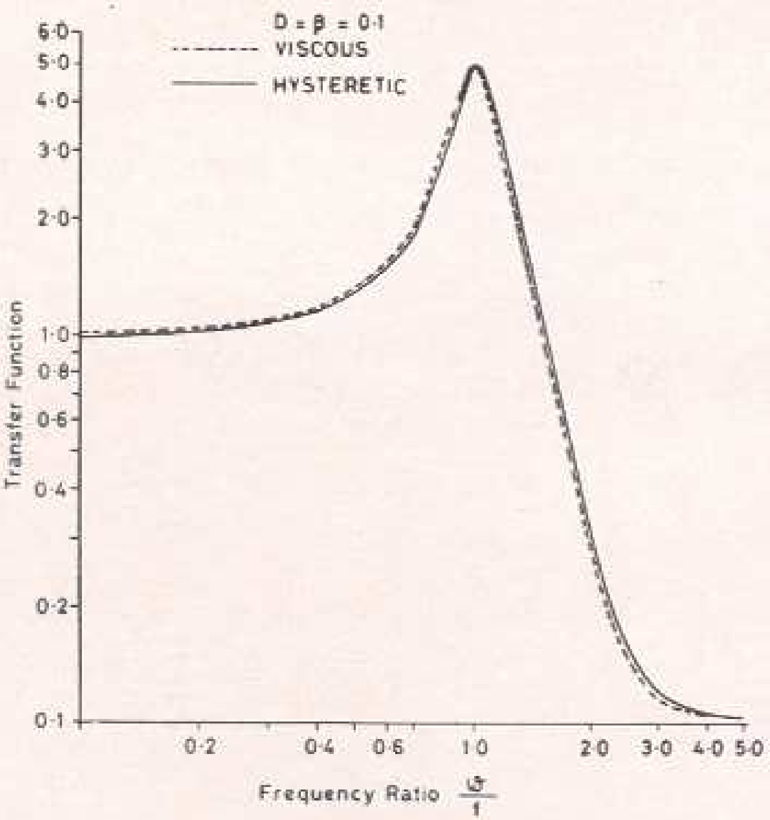


FIG 2a TRANSFER FUNCTION Vs FREQUENCY RATIO (D = β = 0·1)

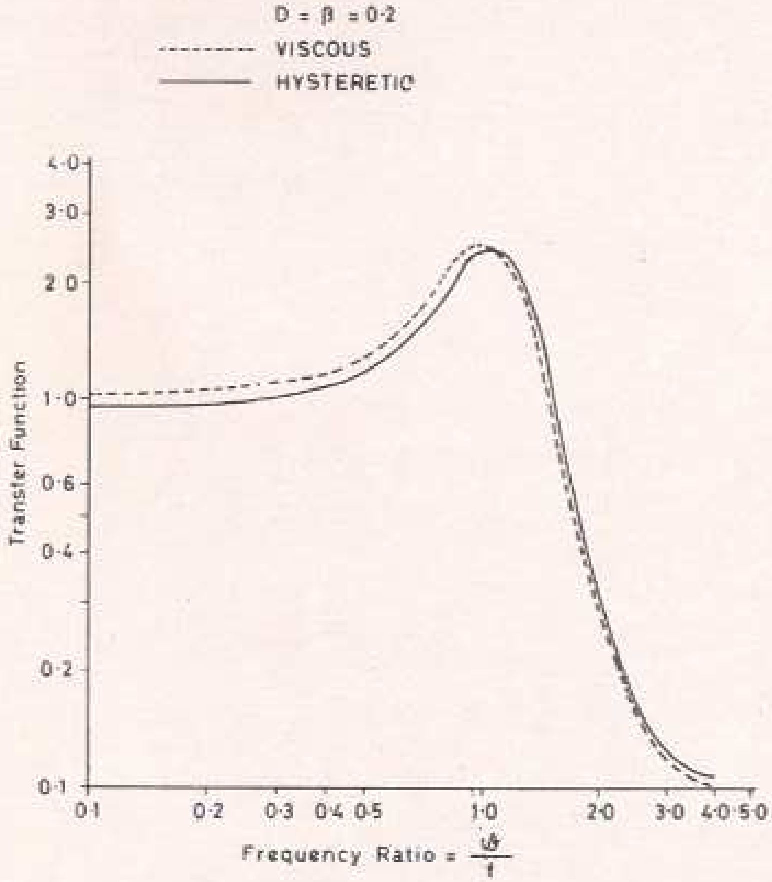


FIG 2b TRANSFER FUNCTION Vs FREQUENCY RATIO (D = β = 0·2)

## TABLE 2 DYNAMIC ANALYSIS OF THE FOUNDATION

| Step | Parameter | Vertical Excitation (Z—Direction) | Horizontal Excitation (X—direction) | Rocking Excitation) (θ—direction) |
|---|---|---|---|---|
| 1 | Mass and Moment of Inertia | $m_z = 153{\cdot}3/9{\cdot}81 = 15{\cdot}6$ t-sec²/m | $m_x = 153{\cdot}3/9{\cdot}81 = 15.6$ t sec²/m | $I_\theta = 137{\cdot}2$ tm—sec² |
| 2 | Spring Stiffnesses: | | | |
| | (a) Equivalent radius $r_o$   $r_o = \sqrt{\frac{BL}{\pi}}$ | $= \sqrt{\frac{3{\cdot}50 \times 6{\cdot}30}{\pi}}$ <br> $= 2{\cdot}65$ m | $r_o = \sqrt{\frac{3{\cdot}50 \times 6{\cdot}30}{\pi}}$ <br> $= 2{\cdot}65$ m | $r_o = \sqrt{\frac{BL}{3\pi}} = \sqrt{\frac{3{\cdot}50 \times 6{\cdot}30}{3\pi}}$ <br> $= 3{\cdot}10$ m |
| | (b) Spring constant (circular footing) | $K_z = 16{\cdot}31 \times 10^4$ t/m | $K_x = 13{\cdot}12 \times 10^4$ t/m | $K_\theta = 122{\cdot}22 \times 10^4$ tm/radian |
| 3 | Damping Ratio | | | |
| | (a) Mass ratio | $B_z = 0{\cdot}74$ | $B_x = 0{\cdot}92$ | $B_\theta = 0{\cdot}64$ |
| | (b) Geometrical damping ratio | $D_{zg} = 0{\cdot}49$ | $D_{xg} = 0{\cdot}30$ | $D_{\theta g} = 0{\cdot}114$ |
| | (c) Internal damping ratio | $D_{zi} = 0{\cdot}06$ | $D_{xi} = 0{\cdot}06$ | $D_{\theta}^{i} = 0{\cdot}06$ |
| | (d) Total damping | $D_z = 0{\cdot}55$ | $D_x = 0{\cdot}36$ | $D_\theta = 0{\cdot}174$ |
| | (e) Damping coefficient   $C = D \times 2\sqrt{KM}$ | $C_z = 0{\cdot}55 \times 2 \times \sqrt{16{\cdot}31 \times 10^4 \times 15{\cdot}6}$ <br> $= 1754{\cdot}62$ t sec⁻¹/m | $C_x = 0{\cdot}36 \times 2 \times \sqrt{13{\cdot}12 \times 10^4 \times 15{\cdot}6}$ <br> $= 1030{\cdot}06$ t sec⁻¹/m | $C_\theta = 0{\cdot}174 \times 2 \times \sqrt{122{\cdot}22 \times 10^4 \times 137{\cdot}2}$ <br> $= 4506{\cdot}38$ tm sec⁻¹/rad |
| 4 | (a) Natural Frequency (Hz) (uncoupled)   $\frac{1}{2\pi}\sqrt{K/M}$ | $f_{uz} = \frac{1}{2\pi}\sqrt{\frac{6{\cdot}31 \times 10^4}{15{\cdot}6}}$ <br> $= 16{\cdot}27$ Hz | $f_{ux} = \frac{1}{2\pi}\sqrt{\frac{13{\cdot}12 \times 10^4}{5{\cdot}6}}$ <br> $= 14{\cdot}59$ Hz | $f_{u\theta} = \frac{1}{2\pi}\sqrt{\frac{122{\cdot}22 \times 10^4}{137{\cdot}2}}$ <br> $= 15{\cdot}02$ Hz |
| | (b) Natural frequency (coupled) | $\omega_{uz} = 102{\cdot}2$ rad/sec | $f_1 = 10{\cdot}66$ Hz <br> $\omega_1 = 67$ rad/sec | $f_2 = 20{\cdot}68$ Hz, <br> $\omega_2 = 128{\cdot}8$ rad/sec |
| | (c) Mode shape (normalised) | | $\phi_1 = \begin{bmatrix} 0{\cdot}21 \\ 0{\cdot}096 \end{bmatrix}$ | $\phi_2 = \begin{bmatrix} 0{\cdot}143 \\ 0{\cdot}141 \end{bmatrix}$ |

*(Contd.)*

Table 2 *Contd*

| *Step* | *Parameter* | | *Vetical Excitation (Z—direction)* | *Horizontal Excitation (X—direction )* | *Rocking Excitation (θ—direction)* |
|---|---|---|---|---|---|
| | (d) Equivalent modal damping ratio | | | $\beta_1 eq = 0{\cdot}1775$ | $\beta_2 eq = 0{\cdot}416$ |
| | (e) Resonance frequency (uncoupled) | $f_r = f/\sqrt{1\text{-}2D^2}$ | $f_{ruz} = 16{\cdot}27/\sqrt{1\text{-}2\times 0{\cdot}55^2}$ $= 41{\cdot}19$ Hz | $f_{rux} = 14{\cdot}59/\sqrt{1\text{-}2\times 0{\cdot}36^2}$ $= 19{\cdot}69$ Hz | $f_{ru} = 15{\cdot}02/\sqrt{1\text{-}2\times 0{\cdot}174^2}$ $= 15{\cdot}99$ Hz |
| | (f) Resonance frequency (coupled) | | | $f_{rx} = 10{\cdot}66/\sqrt{1\text{-}2\times 0{\cdot}1775^2}$ $= 11{\cdot}38$ Hz | $f_{r2} = 20{\cdot}68/\sqrt{1\text{-}2\times 0{\cdot}416^2}$ $= 31{\cdot}63$ Hz |
| 5 | Generated Forces | | $P_z = 3{\cdot}9 \text{ Sin } \omega t$ | $P_x = 2{\cdot}4 \text{ Sin } \omega t$ | $M\theta = 2{\cdot}4\times 2 \text{ Sin } \omega t$ |
| 6 | Transformed Forces to Modal Coordinate | $[\phi]^T P$ | -do- | $P_1 = 0{\cdot}965 \text{ Sin } \omega t$ | $P_2 = 0{\cdot}326 \text{ Sin } \alpha t$ |
| 7 | Modal maximas | $\frac{P}{\sqrt{(p^2 - \alpha^2)^2 + 4\beta^2 \omega^2}}$ | $q_{zmax} = 4{\cdot}01\times 10^{-4}$ | $q_{1max} = 2{\cdot}55\times 10^{-4}$ | $q_{2max} = -2{\cdot}05 = 10^{-5}$ |
| 8 | Displacement Respone | $[\phi][q] = [x]$ | $Z_{max} = 4{\cdot}01\times 10^{-4}$ | $X_{max} = 5{\cdot}06\times 10^{5}$** | $\theta_{max} = 2{\cdot}74\text{x}10\times 10^{-5}$** |
| 9 | Horizontal Amplitude at the Top of Foundation | $x_t = x_{max} + h.\theta_{max}$ | | $x_t = 5{\cdot}06\times 10^{-5} + 1{\cdot}15\times 1{\cdot}37\times 10^{-5}$ $= 6{\cdot}64\times 10^{-5}$ m | $= 6{\cdot}64\times 10^{-2}$ mm $= 0{\cdot}0624$ mm $0{\cdot}3$ mm (OK) |

* The resonance criteria are satisfied as the ratio of the operating speed of machine (4·33 Hz) and the lowest resonant frequency (9·99 Hz) is less than 0·67.

** It is assumed that modal maxima occur at the same time.

## DYNAMIC ANALYSIS OF A CRUSHER FOUNDATION

Dynamic analysis of a Jaw crusher foundation is presented using the concept of equivalent viscous damping explained above. Coupling between horizontal and rocking modes has been considered for finding the natural frequencies and response of the foundation. Machinery characteristics, such as weight of various components and the speed, soil parameters and size of the block foundation are given in Table 1. Ritchart's formulation is used for determining the stiffness and damping coefficients of the soil. The entire calculations for the dynamic analysis are summarised in Table 2.

## CRI EXPERTISE

Necessary expertise is available in CRI for dealing with analysis/design of machine foundations including necessary computer programmes.

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